Questions and Answers I

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Q: what is a relative TFT?

A: an n-dimensional TFT Z relative to an $(n+1)$-dimensional TFT T in the sense of Freed-Teleman is a TFT which takes values in T in the sense that $Z(M^k)$ is compatibly an element of $T(M^k)$. E.g. the value of Z on an n-manifold M^n is not a number, but an element of the vector space $T(M^n)$. Equivalently, Z is a local boundary condition for T: Z is determined by $Z(\text{pt})$ which is an element of $T(\text{pt})$.

Theory X is a TFT relative to a 7d TFT which Freed-Teleman write down. It is not very interesting. Similarly, Chern-Simons theory is a TFT relative to a 4d TFT. This is one way to describe the sense in which Chern-Simons is anomalous. WZW is also a TFT relative to Chern-Simons.

Q: can you elaborate on the case of WZW and Chern-Simons?

A: Recall that $\text{CS}_{G,k}(S^1)$ is the modular tensor category associated to $U_q(\mathfrak{g})$ (where q is some root of unity depending on k) and that $\text{CS}_{G,k}(\Sigma)$ is the space of global sections of L^k where L is a line bundle on $\text{Bun}_G(\Sigma)$.

The boundary condition that WZW gives is related to the following functionals: starting from a global section of L^k on $\text{Bun}_G(\Sigma)$ we can restrict it to the trivial G-bundle, and that gives us an element of a line. Similarly, starting from an object in the modular tensor category we can associate an integrable level k representation of the loop group LG.

It's harder to describe what happens on a point. Andre will tell us later.

Q: what's the early history of Theory X?

A: Nahm's theorem classifying superconformal Lie algebras suggests that 6 is the biggest dimension in which we could expect to find superconformal field theories. One can ask the question of whether there is a 6d superconformal field theory with this supersymmetry. M-theory was a motivation for doing this.

Recall that string theory is supposed to be a refined theory of gravity. D-branes in string theory are boundary conditions describing where strings can end. We can ask for strings starting and ending on a D-brane and take a limit in which gravitational interactions vanish; the result is usually a gauge theory or more generally a QFT, and in particular the metric is now fixed (background) rather than varying (dynamical).

Theory X was discovered by Witten and Strominger around 1995 in the sense that they gave a string-theoretic procedure to perform the above limit. There were some arguments about it for awhile.

Q: what is the flavor of these arguments, for a mathematical audience?

A: there were no Lagrangians around, so the status of what happened was unclear. These days it seems like the strongest arguments come from AdS/CFT. M-theory can be formulated on $\text{AdS}_7 \times S^4$. The symmetries of the first factor can be identified with the ideal boundary of AdS_7, and string theory on this product corresponds to Theory X on the boundary (maybe?).

Q: why do people think the local operators exist?

A: in M-theory it's reasonable to intersect branes. Only some intersections are allowed, e.g. an M2-brane intersecting an M5-brane or an M5-brane intersecting an M5-brane. This is one motivation. Another one is looking at the operators in 5d SYM and trying to lift them.

A mathematical calculation you can perform related to this involves a calculation of possible central charges of central extensions of the relevant super Lie algebras. This tells you what kind of things you have room for.